

## Chapter 7 Notes

\*STUDENT COPY\*

Final Mark: /8

Marks → Requirement ↓	2	1	0
Notes Present	All notes present	Most notes present	Less than half of notes present
Organization / Neatness	Notes in chronological order, name and date on everything	Almost all notes in chronological order, name and date on most pages	Mostly out of order, name and date often missing
Questions	Question column completed on all notes, higher level questions attempted	Most question columns complete, some higher level questions	Less than half of the question columns complete
Main Ideas and Reflections	All 'main ideas' and 'reflections' complete <u>with care</u> in notes	Most 'main ideas' and 'reflections' complete in notes	Less than half of the 'main ideas' and 'reflections' complete

\*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

\*TEACHER COPY\*

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## 7.2 - Solving Systems with Graphs

Name: *Notes key*

Goal: to use graphs to solve linear systems

Date:

### Toolkit:

- graphing lines
- rearranging equations
- substitution

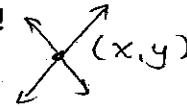
### Summary / Main Ideas:

### Definitions:

**Linear System** - two or more linear equations together is called a **linear system**.

①  
②

**Solving a System** - to solve a linear system, find the coordinates where the two lines intersect (the point where the lines cross). You will have an  $x$ -value and a  $y$ -value!

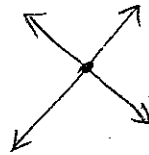


### Steps for solving systems graphically:

1. Change each equation to a form that is easy to graph ( $y = mx + b$  or  $Ax + By = C$ )
2. Graph each line
3. Write the solution (state the point where the lines cross)  $(x, y)$
4. Check the solution by substituting into each original equation (point must "satisfy" both lines)

What are the three possibilities when two lines are graphed?

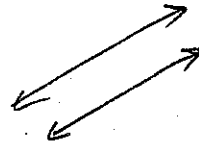
A One solution



(different slopes)

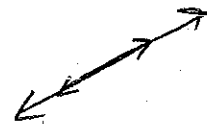
\* Most of our questions have one sol'n! \*

B no solution



Parallel Lines (same slope, different  $y$ -int.)

C infinite solutions



Same line!  
- same slope  
- same  $y$ -int.

Ex1) Solve the system graphically and check the solution

Step 1:

$$\begin{cases} ① x + y = 7 \\ ② 3x + 4y = 24 \end{cases}$$

Step 2:

$$\begin{array}{l} ① x + y = 7 \\ \underline{x\text{-int}} \quad \underline{y\text{-int}} \\ x = 7 \quad y = 7 \end{array}$$

Slope:  $-\frac{1}{1}$

or  $y = -x + 7$   
↑    ↑  
m    b

$$\begin{array}{l} ② 3x + 4y = 24 \\ \underline{x\text{-int}} \quad \underline{y\text{-int}} \\ x = 8 \quad y = 6 \end{array}$$

Slope:  $-\frac{3}{4}$

or  $\frac{4y}{4} = \frac{-3x + 24}{4}$   
 $y = \frac{-3}{4}x + 6$   
②    ①

Step 3:

Solution is  $(4, 3)$

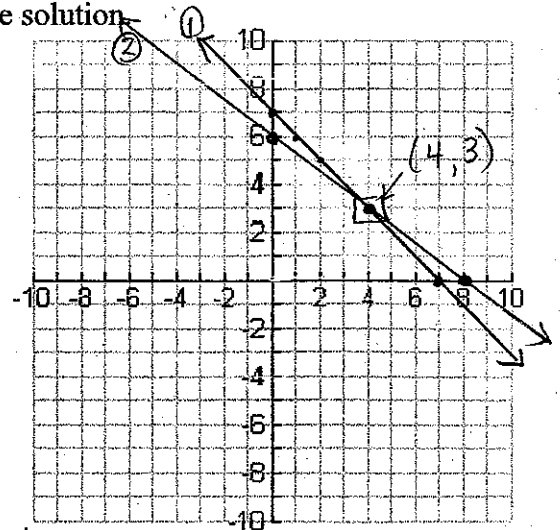
Step 4: Check:

① LS	RS
$x + y$	7
$4 + 3$	
7	

✓

② LS	RS
$3x + 4y$	24
$3(4) + 4(3)$	
$12 + 12$	
24	

✓



What if you just need to "check"?

Ex2) Is  $(2, -1)$  a solution to the following system?

$\begin{array}{l} \text{check:} \\ \text{① LS} \quad   \quad \text{RS} \\ 3x + 5y \\ 3(2) + 5(-1) \\ 6 + -5 \\ 1 \end{array}$	$\begin{array}{l} \text{② LS} \quad   \quad \text{RS} \\ 2x - 2y \\ 2(2) - 2(-1) \\ 4 + 2 \\ 6 \end{array}$	$\begin{array}{l} \text{① } 3x + 5y = 1 \\ \text{② } 2x - 2y = 5 \end{array}$
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$6 \neq 5$

$(2, -1)$  does not satisfy both equations  
 $\therefore$  is not a solution!

Ex3) Solve the system by graphing. Explain whether the solution is exact or approximate.

Step 1:

$$\begin{array}{l} \text{① } x + 2y - 5 = 0 \rightarrow \text{① } x + 2y = 5 \\ \text{② } x - 2y - 13 = 0 \rightarrow \text{② } x - 2y = 13 \end{array}$$

Step 2:

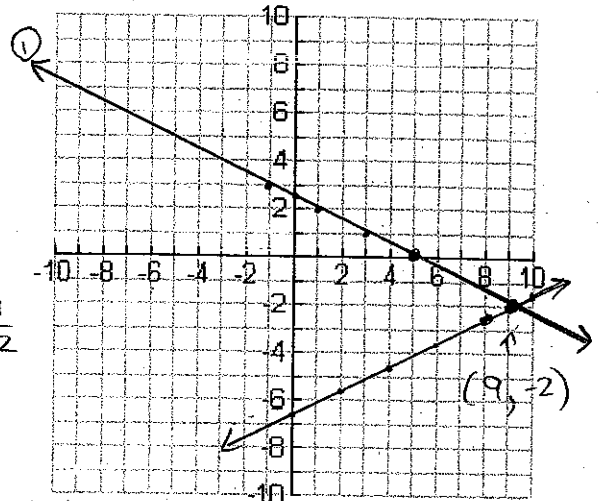
①  $x + 2y = 5$   
 $x\text{-int} = 5$   $y\text{-int} = \frac{5}{2} = 2.5$  slope =  $-\frac{1}{2}$

②  $x - 2y = 13$   
 $x\text{-int} = 13$   $y\text{-int} = -\frac{13}{2} = -6.5$  slope =  $+\frac{1}{2}$

Step 3: solution is  $(9, -2)$

Step 4:

$\begin{array}{l} \text{① LS} \quad   \quad \text{RS} \\ x + 2y \quad   \quad 5 \\ 9 + 2(-2) \\ 9 - 4 \\ 5 \end{array}$	$\begin{array}{l} \text{② LS} \quad   \quad \text{RS} \\ x - 2y \quad   \quad 13 \\ 9 - 2(-2) \\ 9 + 4 \\ 13 \end{array}$
---	---



The solution is exact (but was hard to be sure)

Ex4) Solve the system by graphing. Explain whether the solution is exact or approximate.

①  $y = -2x + 7$   
 ②  $7x - 2y = 0$

Step 1:

①  $y = -2x + 7$  (m = -2, b = 7)

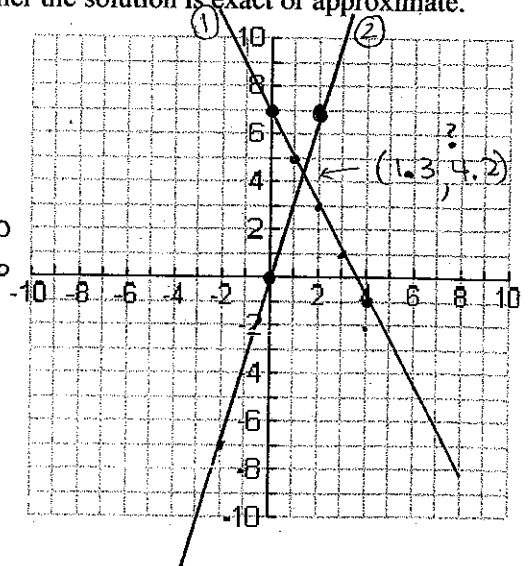
②  $7x - 2y = 0$   
 $-2y = -7x$   
 $\frac{-2y}{-2} = \frac{-7x}{-2}$   
 $y = \frac{7}{2}x + 0$  (m =  $\frac{7}{2}$ , b = 0)

Step 2: ①  $m = -\frac{2}{1}$   $b = 7$

②  $m = \frac{7}{2}$   $b = 0$

Step 3: solution is  $(1.3, 4.2)$

Step 4: the solution is approximate.



Reflection: What is a disadvantage to solving a system using GRAPHING?

- The results may be approximate or hard to see.
- it is difficult with large or awkward numbers.

## 7.4 – Solving Systems Algebraically using Substitution

Name: Notes key  
Date:

**Goal:** to use the substitution of one variable to solve a linear system

### Toolkit:

- rearranging equations
- substituting in to eqns
- a solution is the point  $(x, y)$  where the lines meet  $\therefore$  need  $x$ , need  $y$ !

### Main Ideas:

Linear systems can be solved without graphing. One method is by substitution.

### Steps:

1. Solve one equation for either  $x$  or  $y$  (get either  $x$  or  $y$  by itself).  
Let's say you get  $y$  by itself in this case.
2. Substitute the equation into the second equation
3. Solve the second equation for the other variable (in this case  $x$ )
4. Now that you have the solution to one variable (in this case  $x$ ), substitute it into one of the original two equations to get  $y$ .
5. Write the solution
6. Check that the solution satisfies each equation

Ex1) Solve by substitution and check

$$\textcircled{1} 3x + y = 3 \quad \leftarrow \text{solve for } y!$$

$$\textcircled{2} 7x - 2y = 20$$

step 1: look at both equations. which variable, in which eq'n is easiest to get by itself?  $y$  in  $\textcircled{1}$ .

$$\textcircled{1} \begin{array}{r} 3x + y = 3 \\ -3x \quad -3x \\ \hline \text{"new look"} \textcircled{1} y = -3x + 3 \end{array}$$

step 2:  $\textcircled{2} 7x - 2y = 20$

$$7x - 2(-3x + 3) = 20$$

step 3:  $\underline{7x} + \underline{6x} - 6 = 20$

$$\begin{array}{r} 13x - 6 = 20 \\ +6 \quad +6 \\ \hline 13x = 26 \\ \frac{13}{13} \quad \frac{13}{13} \end{array}$$

$$\textcircled{x = 2}$$

step 4: substitute  $x = 2$  in either original equation

$$\textcircled{1} 3x + y = 3$$

$$3(2) + y = 3$$

$$6 + y = 3$$

$$\begin{array}{r} 6 + y = 3 \\ -6 \quad -6 \\ \hline y = -3 \end{array}$$

step 5: If these lines were graphed, they would intersect at  $(2, -3)$ .

Solution: $(2, -3)$	$x = 2$
	$y = -3$

step 6: check!

$\textcircled{1}$	$3x + y$	$3$	$\left  \begin{array}{l} \text{LS} \\ \text{RS} \end{array} \right.$	$\textcircled{2}$	$7x - 2y$	$20$	$\left  \begin{array}{l} \text{LS} \\ \text{RS} \end{array} \right.$
	$3(2) + (-3)$				$7(2) - 2(-3)$		
	$6 - 3$				$14 + 6$		
	$3$	$\checkmark$			$20$		$\checkmark$

Equations solved for y

Ex2) Solve by substitution

1)  $y = 3x + 2$  step 1: y already by itself in ①

2)  $y = -x - 14$

$$\begin{array}{r} \underline{\underline{2}} \\ 3x + \cancel{y} = -\cancel{y} - 14 \\ \cancel{+y} \quad \cancel{-y} \end{array}$$

$$\underline{\underline{3}} \quad \frac{4x}{4} = \frac{-16}{4}$$

$$x = -4$$

$$\begin{array}{r} \text{①} \\ x = -4 \\ y = 3x + 2 \end{array}$$

$$y = 3(-4) + 2$$

$$y = -12 + 2$$

$$y = -10$$

S Solution:  $(-4, -10)$  or  $x = -4, y = -10$

Equations with fractions

\* Clear fractions! \*

Ex3) Solve by substitution

Clear Fractions!

Multiply all terms by a number (LCM) that will cancel with denominators.

$$\begin{cases} \frac{x}{8} + \frac{y}{8} - \frac{2}{15} = 0 \rightarrow \text{① } -3x + 5y - 2 = 0 \\ \frac{x}{7} + \frac{y}{7} = 0 \rightarrow \text{② } x + 7y = 0 \end{cases}$$

1: get x by itself in ②.  $x = -7y$

$$\underline{\underline{2}} \quad -3x + 5y - 2 = 0$$

$$-3(-7y) + 5y - 2 = 0$$

$$\underline{\underline{3}} \quad 21y + 5y = 2$$

$$\frac{26y}{26} = \frac{2}{26}$$

$$y = \frac{2}{26}$$

$$y = \frac{1}{13}$$

4 sub into eqn ②

$$x + 7y = 0$$

$$x + 7\left(\frac{1}{13}\right) = 0$$

$$x + \frac{7}{13} = 0$$

$$\begin{array}{r} -\frac{7}{13} \\ x + \frac{7}{13} = 0 \\ \hline x = -\frac{7}{13} \end{array}$$

Solution:  
 $\left(-\frac{7}{13}, \frac{1}{13}\right)$   
or  $x = -\frac{7}{13}, y = \frac{1}{13}$

**Reflection:** When you have a system with fractions in it, and you want to write an equivalent system without fractions, how do you decide what number to multiply by?

# 7.5 - Solving Systems Algebraically using Elimination

Name: Notes  
Date: Key

**Goal:** to use the elimination of one variable to solve a linear system

**Toolkit:**

- substitution
- rearranging eq'ns
- coefficient: the number multiplied by a variable  
e.g.  $2x \rightarrow$  coefficient of  $x$  is  $2$ .

**Main Ideas:**

Linear systems can be solved without graphing. One method is by elimination.

**Steps:**

1. *\*May not be necessary\** Multiply both sides of one or both equations by a constant to get either the same  $x$  or the same  $y$  coefficient in both equations *\*you get an "equivalent system"\**
2. Add or subtract the two equations to eliminate either  $x$  or  $y$
3. Solve the resulting equation for the remaining variable
4. Substitute the value obtained in step 3 back into one of the original equations to get the other variable
5. Write the solution
6. Check that the solution satisfies each equation

Ex 1) Solve the system by elimination and check

$$\begin{array}{l} 1) 3x - 5y = -9 \\ 2) 4x + 5y = 23 \end{array}$$

Step 1: not needed (the  $y$ 's can be eliminated by adding the eq's)

Step 2:

$$\begin{array}{r} 1) 3x - 5y = -9 \\ + 2) 4x + 5y = 23 \\ \hline 7x = 14 \end{array}$$

Step 5: Solution  $(2, 3)$   
or  $x=2, y=3$

Step 3:

$$\begin{array}{r} 7x = 14 \\ \hline x = 2 \end{array}$$

Step 4: sub in  $x=2$  to either eq'n 1 or 2

$$\begin{array}{l} 1) 3x - 5y = -9 \\ 3(2) - 5y = -9 \\ 6 - 5y = -9 \\ -5y = -15 \\ \hline y = 3 \end{array}$$

Step 6: Check

1) LS	RS	2) LS	RS
$3x - 5y$	$-9$	$4x + 5y$	$23$
$3(2) - 5(3)$		$4(2) + 5(3)$	
$6 - 15$		$8 + 15$	
$-9$	✓	$23$	✓

How do you know when to add the eq'ns or subtract the eq'ns in step 2?

If the variable you are eliminating has the same coefficient (but with different signs), you must add.

e.g.

$$\begin{array}{r} -5y \\ + 5y \\ \hline 0 \end{array}$$

If the coeff's are exactly the same (same sign, too) you subtract

e.g.

$$\begin{array}{r} +2y \\ - 2y \\ \hline 0 \end{array}$$

same 4x  
sign 4x  
subtract!

Ex 2) Solve by elimination step not needed.

$$\begin{array}{l} 1) 4x + 3y = 5 \\ 2) 4x - 7y = 15 \end{array} \rightarrow \begin{array}{l} 4x + 3y = 5 \\ - (4x - 7y = 15) \\ \hline 10y = -10 \\ y = -1 \end{array}$$

$$\begin{array}{l} 4) \textcircled{1} 4x + 3y = 5 \\ 4x + 3(-1) = 5 \\ 4x - 3 = 5 \\ 4x = 8 \\ x = 2 \end{array}$$

S Solution:  $(2, -1)$  or  $x=2, y=-1$

Ex 3) Solve by elimination step 1 needed! Must match either x's or y's

$$\begin{array}{l} 1) 2x + 5y = 11 \rightarrow \times 3 \\ 2) 3x - 2y = 7 \rightarrow \times 2 \end{array} \rightarrow \begin{array}{l} 6x + 15y = 33 \\ - (6x - 4y = 14) \\ \hline 19y = 19 \\ y = 1 \end{array}$$

$33 - 14 = 19$

run equation through "bars" and multiply every term by what's there.

$$\begin{array}{l} 4) \textcircled{1} 2x + 5y = 11 \\ 2x + 5(1) = 11 \\ 2x + 5 = 11 \\ 2x = 6 \\ x = 3 \end{array}$$

S Solution:  $(3, 1)$  or  $x=3, y=1$

Equations with fractions

- \* clear fractions
- \* how could you clear decimals? ( $\times 10, \times 100, \text{etc.}$ )

Ex 4) Solve by elimination  
Clear fractions!

$$\begin{array}{l} 1) \left\{ \begin{array}{l} \frac{3}{4}x - y = 2 \\ \frac{1}{8}x + \frac{1}{4}y = 2 \end{array} \right\} \times 4 \rightarrow \begin{array}{l} 3x - 4y = 8 \\ x + 2y = 16 \end{array} \\ 2) \left\{ \begin{array}{l} \frac{3}{4}x - y = 2 \\ \frac{1}{8}x + \frac{1}{4}y = 2 \end{array} \right\} \times 8 \rightarrow \begin{array}{l} 3x - 4y = 8 \\ 2x + 4y = 32 \end{array} \end{array}$$

$$\begin{array}{r} 3x - 4y = 8 \\ + (2x + 4y = 32) \\ \hline 5x = 40 \\ x = 8 \end{array}$$

sub in  $x=8$  to any eq'n

$$\begin{array}{l} x + 2y = 16 \\ (8) + 2y = 16 \\ -8 \\ \hline 2y = 8 \\ y = 4 \end{array}$$

S Solution:  $(8, 4)$  or  $x=8, y=4$

can check to verify.

Reflection: Which method do you prefer for solving linear systems AND WHY: graphing, substitution, or elimination?

# 7.6 – Properties of Systems

Name: Notes  
Date: Key

**Goal:** to recognize systems that will have each of the three different types of solutions

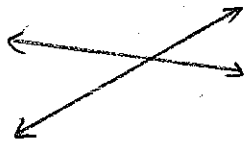
**Toolkit:**

- So far, all of the linear systems we've solved have given one solution (one intersection)
- Rearranging equations

**Main Ideas:**

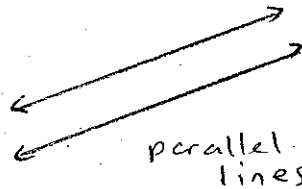
Three types of solutions:

**A ONE solution**  
Sketch:



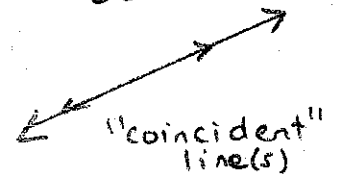
Description:  
- different slopes

**B NO solution**



- same slope  
- different y-intercept

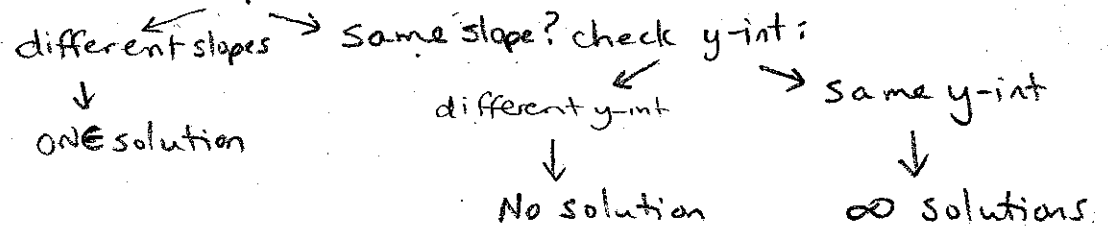
**C Infinite solutions**  
 $\infty$



- same slope  
- same y-intercept (or any other pt.)

How can you predict how many solutions a system will have without graphing?

Check Slopes:



Ex1) Predict how many solutions each system has:

a)  $\begin{cases} ① y = 2x + 3 \\ ② y = \frac{6}{3}x + 3 \end{cases}$

①  $m = 2$   $b = 3$

②  $m = \frac{6}{3} = 2$   $b = 3$

same slope  
same y-int  
 $\therefore$   
 $\infty$  many solutions

b)  $\begin{cases} ① x - y = 4 \\ ② 6x + y = 5 \end{cases}$

①  $m = +\frac{1}{1}$

②  $m = -\frac{1}{1}$

different slopes  
 $\therefore$   
one solution

c)  $\begin{cases} y = -\frac{1}{2}x + 7 \\ y = -\frac{1}{2}x + 2 \end{cases}$

same slope different y-int  
 $\therefore$   
no solution



Standard form "shortcut": start off like elimination—try to get  $x$  or  $y$  coefficients to match by multiplying the whole equation by a constant

Ex 2) How many solutions?  $\begin{cases} 2x - 5y = 15 \\ 4x - 10y = 6 \end{cases} \xrightarrow{\times 2} \begin{cases} 4x - 10y = 30 \\ 4x - 10y = 6 \end{cases}$  not a match  
 match match NO solution!

A) If  $x$  and  $y$  coefficients DO NOT BOTH match, then you have ONE solution

B) If  $x$  and  $y$  coefficients BOTH match, but the constants DO NOT, then you have NO solution

C) If  $x$  and  $y$  coefficients BOTH match, and the constants match, then you have INFINITE solutions (all 3 numbers are the same)

Ex 3) How many solutions does each system have?

a)  $\begin{cases} 7x - y = 10 \\ 14x - 2y = 20 \end{cases} \xrightarrow{\times 2} \begin{cases} 14x - 2y = 20 \\ 14x - 2y = 20 \end{cases}$  all 3 match (same slope, same y-int)  
 $\therefore \infty$  solutions

b)  $\begin{cases} 4x - 3y = 12 \\ 8x - 6y = 30 \end{cases} \xrightarrow{\times 2} \begin{cases} 8x - 6y = 24 \\ 8x - 6y = 30 \end{cases}$   $x$ - and  $y$ -match (same slope) but constant doesn't  
 $\therefore$  NO solution

c)  $\begin{cases} 5x + y = 16 \\ 2x - 3y = 3 \end{cases} \xrightarrow{\times -3} \begin{cases} -15x - 3y = -48 \\ 2x - 3y = 3 \end{cases}$  different  $x$ -coefficients (diff. slopes)  
 $\therefore$  ONE solution.

**Reflection:** Use examples and/or diagrams to explain why there cannot be exactly 2 solutions to a linear system.

To cross only 2 times, a curve would be needed, then it would not be linear anymore.



# 7.7 - Applications of Systems Part I

Name: Notes  
Date: Key

**Goal:** to model situations and answer problems using a system of linear equations

**Toolkit:**

- <sup>total</sup> sum, greater than is +
- difference, less than is -
- times, product is ×
- to change % to dec, move decimal two places to the left
- remember units!

**Main Ideas:**

*These word problems involve two unknowns. We need two equations to solve for two unknowns, so it will be your job to create the system of two equations and solve it!*

**Steps:**

1. Define your two variables. You may use  $x$  and  $y$ , but it is also good to practise working with other variables (such as  $t$  for time). Use "let" statements (e.g. let  $x$  be the number of...).  
*Usually, they are the two things you need in order to answer the problem.*
2. Build your two equations.
3. Solve the system using elimination, substitution, or graphing.
4. Write a sentence answer.
5. Check.

Ex 1) <sup>1</sup>The sum of two numbers is 53. <sup>2</sup>The first is 7 greater than the second.  
 $+ \quad \quad \quad = 53$  // first = add 7 to second

What are the numbers?

Step 1: let  $x$  be the "first number" Let  $y$  be the "second #"

Step 2: From sentence 1: sum (+) of #s  $x, y$  is (=) 53.

①  $x + y = 53$

From sentence 2: first ( $x$ ) is (=) 7 more (+7) than second ( $y$ )

②  $x = y + 7$

Step 3: Solve! use substitution (since eq'n ② already solved for  $x$  ...)

$x = y + 7$

①  $x + y = 53$

$y + 7 + y = 53$

$2y + 7 = 53$

$2y = 46$

$y = 23$

→ sub back in for  $x$ :

$x = y + 7$

$x = 23 + 7$

$x = 30$

Step 4: The two numbers are 30 and 23.

Step 5: Is 30 7 more than 23? Yes!  
Is 30 + 23 equal to 53? Yes!

can I solve this with elimination?

Ex 2) For a basketball game, 1600 tickets were sold. Some tickets cost \$3 and the rest cost \$2. If the total receipts were \$4000, how many of each kind were sold?

1 Let  $x = \#$  of 3-dollar tickets      Let  $y = \#$  of 2-dollar tickets.  
 2 "1600 tickets sold"  $\rightarrow$  ①  $x + y = 1600$  (check units  $\rightarrow$  tickets + tickets = tickets ✓)

"total receipts \$4000"  $\rightarrow$  need money + money = money:  
 ②  $3x + 2y = 4000$   
 ↑ money from x      ↑ money from y

3 Solve: try elimination

$$\begin{array}{r|l} x + y = 1600 & 2 \times 2x + 2y = 3200 \\ 3x + 2y = 4000 & \rightarrow \ominus \text{ } 3x + 2y = 4000 \\ \hline -x & = -800 \\ \hline x = 800 & \end{array}$$

$$\begin{array}{r} x + y = 1600 \\ 800 + y = 1600 \\ \hline -800 \quad -800 \\ \hline y = 800 \end{array}$$

4 There were 800 3-dollar tickets and 800 2-dollar tickets sold.

Ex 3) Isaac borrowed \$2100 for his college tuition. Part of it he borrowed from a government student fund at 5% annual interest. The rest he borrowed from a bank at 6.5% annual interest. If the total annual interest is \$114, how much did he borrow from each source?

1 Let  $g =$  amount from gov't      Let  $b =$  amt from bank.  
 2 ①  $g + b = 2100$       ②  $0.05g + 0.065b = 114$  ← total interest  
 ↑ gov't interest rate × gov't amount      ↑ bank interest rate × bank amt

3 Elimination  $\rightarrow$  big #s! Try subst.

$$\begin{aligned} \textcircled{1} \quad g &= 2100 - b \\ \textcircled{2} \quad 0.05g + 0.065b &= 114 \\ 0.05(2100 - b) + 0.065b &= 114 \\ 105 - 0.05b + 0.065b &= 114 \\ 105 + 0.015b &= 114 \\ -105 & \quad -105 \\ \hline 0.015b &= 9 \\ \hline b &= 600 \end{aligned}$$

Sub back in  $b = 600$   
 $g = 2100 - b$   
 $g = 2100 - 600$   
 $g = 1500$

4 Isaac borrowed \$1500 from the government and \$600 from the bank.

Reflection: Would you ever need to solve for 3 variables? Think of a scenario and (no need to solve!) explain WHAT you would need in order to be able to solve for 3 variables.

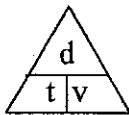
7.8 – Applications of Systems Part II

Name: Notes  
Date: Key

Goal: to continue to model situations and answer problems using a system of linear equations

**Toolkit:**

- sum (+), difference (-), product (×)
- to change % to dec, move decimal two places to the left
- $speed = \frac{dist.}{time}$  OR  $(tv \text{ in the basement})$
- remember units



**Main Ideas:**

These word problems involve two unknowns. We need two equations to solve for two unknowns, so it will be your job to create the system of two equations and solve it!

Same! Steps:

1. Define your two variables. You may use  $x$  and  $y$ , but it is also good to practise working with other variables (such as  $t$  for time). Use "let" statements (e.g. let  $x$  be the number of...).  
*Usually, they are the two things you need in order to answer the problem.*
2. Build your two equations.
3. Solve the system using elimination, substitution, or graphing.
4. Write a sentence answer.
5. Check.

Ex 1)  $P = 2l + 2w$   
The perimeter of a rectangle is 46 cm. What are its dimensions if the length is 4cm less than twice the width?  
*l and w?*

Let  $l = \text{length}$  and  $w = \text{width}$ .

①  $2l + 2w = 46$  (note:  $cm \div 2 \rightarrow l + w = 23$ )

②  $l = 2w - 4$   
"length is" "twice width" "4 less than"

②  $l = 2w - 4$

4 The dimensions are 9cm by 14cm.

①  $l + w = 23$

$2w - 4 + w = 23$

$3w - 4 = 23$

$3w = 27$

$w = 9$

$l = 2w - 4$

$l = 2(9) - 4$

$l = 18 - 4$

$l = 14$

5 check:  $P = 2 \times 9 + 2 \times 14$   
 $= 18 + 28$   
 $= 46 \checkmark$

what else is like:

with "and"  
"against" the  
wind?

- river current
- uphill/downhill

Ex 2) Flying with the wind (faster!) an airplane travels 4256km in 3.5h. Flying against the (slower!) same wind, the airplane makes the return trip (same dist. t) in 3.8h. Find the speed of the airplane in still air and the speed of the wind (assume both speeds are constant for the round trip).

Whenever you're doing a word problem with speed, distance, and time, it helps to set up a table like the one below:

Let  $a$  = speed of airplane

Let  $w$  = speed of wind

$$\text{Speed} = \frac{\text{dist}}{\text{time}}$$

Direction	Distance (km)	Speed (km/h)	Time (h)	Equations
<u>With the wind</u> (add)	4256	$a + w$ same direction	3.5	$a + w = \frac{4256}{3.5}$
<u>Against the wind</u> (subtract)	4256	$a - w$ opposite dir.	3.8	$a - w = \frac{4256}{3.8}$

2 Equations from last column:

$$\textcircled{1} \quad a + w = \frac{4256}{3.5} \rightarrow a + w = 1216$$

$$\textcircled{2} \quad a - w = \frac{4256}{3.8} \rightarrow a - w = 1120$$

3 Elimination:

$\oplus$

$$\frac{2a = 2336}{2 \quad 2}$$

$$\textcircled{a = 1168}$$

Sub back in:

$$a + w = 1216$$

$$1168 + w = 1216$$

$$-1168 \quad -1168$$

$$\textcircled{w = 48}$$

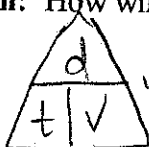
\* Units! \*

4

The speed of the airplane (in still air) was 1168 km/h and the speed of the wind was 48 km/h.

Reflection: How will YOU remember the relationship among distance, speed, and time?

①



"t/v basement"

② Speed = dist/time  
like km/h

③ memorize!

④ others?

